Fundamentals of Dynamic Driven Pile Analysis

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CONTENT

• Introduction
• Why, when and where wave equation
• Wave Equation Models: Hammer, Pile, Soil
• An Example
• Summary
GRLWEAP Application

• **WHEN?**
  – Before pile driving begins, based on estimates
  – After initial pile tests using test results (refined WE)

• **WHY?**
  – Formulate driving criterion (for required capacity)
  – Equipment (hammer and driving system) selection
  – Pile size/impedance selection
  – Stress and blow count calculation (driveability)
  – Capacity determination
GRLWEAP Application: During Design

Get Factored Loads
\[ Q_{ft} = \Sigma (f_i Q_i) \]
Do Borings

Decide Pile Type
Ru Verification and Resistance Factor, \( \phi \)

Required \( R_n = \left( \frac{1}{\phi} \right) Q_{ft} \)
Do Static Analysis
Find Pile Length

Dynamic Analysis: Driveable Pile?

Perform required initial tests

Satisfactory?

Establish Driving Criterion
Drive Production Piles to Blow Count
Test as many as required

No
Wave Equation Application: Capacity Determination by Bearing Graph

**GRLWEAP Fundamentals**

Wave Equation Application: Capacity Determination by Bearing Graph

**GRL Engineers, Inc.**  
**GRLWEAP Example**  
08-Aug-2012  
**GRLWEAP Version 2010**

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**GRLWEAP Example**

- **Ultimate Capacity (kN)**
  - 0
  - 60
  - 120
  - 180
  - 240
  - 300
  - 360
  - 400
  - 800
  - 1600
  - 2400
  - 3200
  - 4000

- **Blow Count (blows/.30m)**
  - 0
  - 60
  - 120
  - 180
  - 240
  - 300
  - 360

- **Compressive Stress (MPa)**
  - 0
  - 4
  - 8
  - 12
  - 16
  - 20

- **Tension Stress (MPa)**
  - 0
  - 4
  - 8
  - 12
  - 16
  - 20

**DELMA D 30-32**

- **Ram Weight** 29.37 kN
- **Efficiency** 0.800
- **Pressure** 9645 (99%) kPa
- **Helmet Weight** 17.79 kN
- **Hammer Cushion** 19259 kN/mm
- **Pile Cushion** 1009 kN/mm
- **COR of P.C.** 0.500
- **Skin Quake** 2.500 mm
- **Toe Quake** 10.160 mm
- **Skin Damping** 0.650 sec/m
- **Toe Damping** 0.500 sec/m
- **Pile Length** 20.50 m
- **Pile Penetration** 14.01 m
- **Pile Top Area** 3716.12 cm²

**Pile Model**

- **Skin Friction Distribution**

**Res. Shaft = 71%**  
(Proportional)
GRLWEAP Application: Driveability Check

- Blow Count < Acceptable
- Stresses < Acceptable
Dynamic Formulas

1893 Wellington (Engineering News)
Now: Modified Gates

Energy Dissipated in Soil = Energy Provided by Hammer

\((R_{UC})(\text{Pile Set}) = (\text{Hammer Efficiency}) \cdot (W_r \cdot h)\)
ENR and Gates for D 19-42

$E_r = 42 \text{ kip-ft} = 57 \text{ kN-m}$

![Graph showing the relationship between blows per 0.25 m and capacity in kN for Gates with calculated stroke and ENR with $Ru = Rd \times 2$.](image)
Shortcomings of Formulas

- Rigid pile model
- Poor hammer representation
- Inherently inaccurate for both capacity and blow count predictions
- No stress results
- Unknown hammer energy
- Relies on EOD Blow Counts
The 1-D Wave Equation

\[ \rho \left( \frac{\partial^2 u}{\partial t^2} \right) = E \left( \frac{\partial^2 u}{\partial x^2} \right) \]

- \( E \): elastic modulus
- \( \rho \): mass density

with \( c^2 = \frac{E}{\rho} \) ... Wave Speed

Solution: \( u = f(x-ct) + g(x+ct) \)

- \( x \): length coordinate
- \( t \): time
- \( u \): displacement
THE WAVE EQUATION MODEL

• The Wave Equation Analysis calculates the displacement of any point of a slender elastic rod at any time using a difference method.

• The calculation is based on rod
  – Length
  – Cross Sectional Area
  – Elastic Modulus
  – Mass density
For a pile driving analysis, the “slender, elastic rod” consists of Hammer+Driving System+Pile.

The soil is represented by resistance forces acting on the pile and representing the forces in the pile-soil interface.
The Pile Model

To solve the wave equation numerically:

- The hammer-pile system is divided into segments (masses and springs)
  - of length $\Delta L$
    - typically: $\Delta L \leq 1 \text{ m (3.3 ft)}$
    - with mass $m = \rho A \Delta L$
    - and stiffness $k = E A / \Delta L$
    - there are $N = L / \Delta L$ pile segments
We can model 3 hammer-pile systems.

(a) Schematic of System  (b) Model

Air/Steam/Hydraulic
Assembly

(c) Soil Model

Pile
Soil

(d) Representation of Soil Model

Dynamic Resistance
Static Resistance

![Graphs showing relationships between velocity, displacement, and dynamic resistance.]

- Hammer Cushion
- Helmet
- Pile Cushion
- Elastic Connection
- Vibrator and Jaws
- Vibration Isolator

GRLWEAP Fundamentals
External Combustion Hammer Modeling

Cylinder and upper frame = assembly top mass

Ram guides for assembly stiffness

Drop height

Ram: A, L for stiffness, mass

Hammer base = assembly bottom mass
External Combustion Hammers
Ram Model

Ram segments
~1m long

Combined Ram-
H.Cushion
Helmet mass
External Combustion Hammers
Combined Ram Assembly Model

Ram segments

Assembly segments

Combined Ram-H.Cushion

Helmet mass
External Combustion Hammer
Analysis Procedure

• Static equilibrium analysis

• Dynamic analysis starts when ram is within 1 ms of impact.

• All ram segments then have velocity

\[ V_{\text{RAM}} = (2g h \eta)^{1/2} - 0.001 g \]

\( g \) is the gravitational acceleration
\( h \) is the equivalent hammer stroke and \( \eta \) is the hammer efficiency
\( h = \text{Hammer potential energy}/\text{Ram weight} \)
External Combustion Hammer Analysis Procedure

• Dynamic analysis ends when
  – Pile toe has rebounded to 80% of max $d_{toe}$
  – Pile has penetrated more than 4 inches
  – Pile toe has rebounded to 98% of max $d_{toe}$ and energy in pile is essentially dissipated
Diesel Hammers

Closed Ended

Open Ended
**Diesel Hammer Components**

- **Piston** = Ram

**Components:**
- Cylinder
- Fuel pump
- Port (closed by piston)
- Compressive stroke
- Combustion chamber
- Impact block
- Hammer Cushion; Helmet
Diesel Hammer Model

Ram segments ~1m long

Ram bottom/impact block

Impact Block mass

Hammer Cushion

Helmet mass
Diesel Hammer Combustion Pressure Model

- Compressive Stroke, $h_c$
- Cylinder Area, $A_{CH}$
- Final Chamber Volume, $V_{CH}$
- Max. Pressure, $p_{MAX}$

Precompression - Combustion - Expansion - Pressure
DIESEL PRESSURE MODEL

Liquid Injection Hammers

Liquid Injection Timing Parameters:
- Combustion Delay, $\Delta t$
- Combustion Duration, $t_D$

Compression:
$p = p_{\text{atm}} (V_{\text{in}} / V)^{1.35}$

Expansion:
$p = p_{\text{MAX}} (V_{CH} / V)^{1.25}$

Time

Pressure

Port Closure

Impact

$\Delta t$

$t_D$

Port

Open
Energy, Efficiency and Measurements
Potential (Rated) → Kinetic → Transferred
Measured Transferred Energy

Max $E_T = \int F(t) \, v(t) \, dt$

(EMX, ENTHRU)

$\eta_T = \frac{ENTHRU}{E_R}$

(transfer ratio or efficiency)

$E_R = W_R \, h$

Manufacturer’s Rating

Measure

Force, $F(t)$

Velocity, $v(t)$

$h$
Measured Transfer Ratios for Diesels

**Steel Piles**

![Graph showing measured transfer ratios for steel piles]

**Concrete Piles**

![Graph showing measured transfer ratios for concrete piles]

**ALL DIESEL HAMMERS ON STEEL**

- $N = 1419$; Median = 38.5%

**ALL DIESEL HAMMERS ON CONCRETE/WOOD**

- $N = 668$; Median = 25.5%

- **Mean = 38.1%**
- **Standard Deviation = 10.1%**

- **Mean = 26.8%**
- **Standard Deviation = 7.8%**
SA Air Hammers – SA Hydraulic Hammers on Steel Piles

SA AIR/STEAM HAMMERS ON STEEL
N = 747; MEDIAN = 55.9%

- Energy transfer ratio (EMx/E-rated)
- Percentile distribution
- Mean: 55.7%
- Standard deviation: 12.7%

SA HYDRAULIC HAMMERS ON STEEL
N = 203; MEDIAN = 69.1%

- Energy transfer ratio (EMx/E-rated)
- Percentile distribution
- Mean: 68.7%
- Standard deviation: 16.3%
GRLWEAP

Impact Hammer Efficiencies, $\eta_h$

*Diesel hammers*: 0.80
*Traditional air/steam hammers*: 0.67
*Hydraulic hammers*: 0.80
*Hammers with energy monitoring*: 0.95

These efficiencies account for energy losses that we cannot calculate or otherwise assess!
Vibratory Hammers
Vibratory Hammer Model

- Line Force
- Bias Mass and
- Oscillator mass, \( m_2 \)
- Eccentric masses, \( m_e \), radii, \( r_e \)
- Clamp

Vibratory Force:
\[
F_V = m_e \left[ \omega^2 r_e \sin \omega t - a_2(t) \right]
\]
Driving System Models

The Driving Systems Consists of

1. Helmet including inserts to align hammer and pile
2. Optionally: Hammer Cushion to protect hammer
3. For Concrete Piles: Softwood Cushion
Driving System Model
Example of a diesel hammer on a concrete pile

Hammer Cushion: Spring plus Dashpot

Helmet + Inserts

Pile Cushion + Pile Top: Spring + Dashpot

Pile Top Mass
The Soil Model

After Smith

Outer Soil:
Rigid

Interface Soil:
Elasto-Plastic Springs and Viscous Dashpots
Soil Resistance

- Soil resistance slows pile movement and causes pile rebound
- A very slowly moving pile only encounters static resistance
- A rapidly moving pile also encounters dynamic resistance
- The static resistance to driving differs from the soil resistance under static loads
Smith’s Soil Model

Total Soil Resistance

\[ R_{\text{total}} = R_{si} + R_{di} \]

- **Segment** \( i \)
  - Displacement \( u_i \)
  - Velocity \( v_i \)

- **Fixed**
  - \( R_{si} \) function of \( u_i \)
  - \( R_{di} \) function of \( v_i \)
Static Shaft Resistance

Model Parameters $R_{ui}$, $q_i$

Elastic spring with max. compression $q$ (quake)

Rigid plastic slider with Resistance $R_{ui}$

$R_{ui}$

$k_{si} = R_{ui} / q_i$

quake, $q_i$

Fixed reference

Pile Segment

Pile Displacement
Shaft Resistance and Quake

\[ -R_{ui} \]

\[ q_i \]

Static Resistance

Recommended Shaft Quakes:

- 2.5 mm or
- 0.1 inches

Pile Displacement
The Static Toe Resistance and Quake

For impact hammers

For vibratory hammers

Static Toe Resistance

Toe Displacement
Recommended Toe Quakes, $q_t$

**Non-displacement piles**
- 0.1” or 2.5 mm
- 0.04” or 1 mm on hard rock

**Displacement piles**
- D/120: very dense/hard soils
- D/60: softer/loose soils
Smith’s Soil Damping Model (Shaft or Toe)

- **Pile Segment**
- **Fixed reference (soil around pile)**
- **Velocity v**
- **Dashpot**

**Smith damping factor**, \( J_s \) [s/m or s/ft]

\[
R_d = R_s J_s v
\]

**Smith-viscous damping factor**, \( J_{sv} \) [s/m or s/ft]

\[
R_d = R_u J_{sv} v
\]

For RSA and Vibratory Analysis
Recommended Smith damping factors

**Shaft**
- Clay: $0.65 \text{ s/m or } 0.20 \text{ s/ft}$
- Sand: $0.16 \text{ s/m or } 0.05 \text{ s/ft}$
- Silts: use an intermediate value
- Layered soils: use a weighted average for bearing graph

**Toe**
- All soils: $0.50 \text{ s/m or } 0.15 \text{ s/ft}$
Resistance Distribution

1. Simplest
   I. Percentage Shaft resistance (from static soils analysis)
   II. Triangular or Rectangular or Trapezoidal

   Only reasonable for a simple Bearing Graph where little is known about soil.

   End Bearing = Total Capacity x (100% - Percent Shaft Resistance)
Resistence Distribution

2. Still Simple:

ST Analysis based on some knowledge of Soil Types

Reasonable for a simple Bearing Graph; for Driveability possible, but more accurate analysis should be done.

End Bearing = From Soil Type, Pile Bottom Area
Resistance Distribution

3. More Involved:

I. SA
   Input: SPT Blow Count, Friction Angle or Unconfined Compressive Strength

II. API (offshore wave version)
   Input: Friction Angle or Undrained Shear Strength

III. CPT
   Input: Cone Record including Tip Resistance and Sleeve Friction vs Depth.

All are good for a Bearing Graph
May be OK for Driveability Analysis
Local experience may provide better values
Numerical Treatment

- **Predict displacements:**
  \[ u_{ni} = u_{oi} + v_{oi} \Delta t \]

- **Calculate spring compression:**
  \[ c_i = u_{ni} - u_{ni-1} \]

- **Calculate spring forces:**
  \[ F_i = k_i c_i \]

- **Calculate resistance forces:**
  \[ R_i = R_{si} + R_{di} \]
**Force balance at a segment**

**Force from upper spring,** $F_i$

**Resistance force,** $R_i$

**Force from lower spring,** $F_{i+1}$

Velocity, $v_i$, and Displacement, $u_i$, from Integration

**Acceleration:**
$$a_i = \frac{(F_i + W_i - R_i - F_{i+1})}{m_i}$$
Set or Blow Count Calculation

(a) Simplified: extrapolated toe displacement

**Static soil Resistance**

- **Max. Displacement**
- **Calculated**
- **Extrapolated**
- **Pile Displacement**
- **Final Set**
- **Quake**

$R_u$
(b) Blow Count Calculation by RSA

- Residual Stress Analysis is also called Multiple Blow Analysis
- Analyzes several blows consecutively with initial stresses, displacements from static state at end of previous blow
- Yields residual stresses in pile at end of blow; generally lower blow counts
Blow Count Calculation (b) Residual Stress Analysis (RSA)

Set for 2 Blows

Convergence: Consecutive Blows have same pile compression/sets
RSA: When and How

• RSA is the preferable method of analysis for long slender piles (e.g., steel piles with \(L_p/D > 100\) or Monotube/Taper Tube piles)

• RSA calculates somewhat higher blow counts than standard analysis (non-conservative)

• RSA calculates somewhat higher stresses than standard analysis (conservative)
Program Flow – Bearing Graph

Input

Model hammer, driving system and pile

Choose first Ru

Distribute Ru
Set Soil Constants

Static Equilibrium
Ram velocity
Dynamic analysis

• Pile stresses
• Energy transfer
• Pile velocities

Calculate Blow Count

Increase Ru

Increase Ru?

Output
Bearing Graph: Variable Capacity, One depth
SI-Units; Clay and Sand Example; D19-42; HP 12x53;
The Inspectors’ Chart:
One Capacity and One Depth – Stroke Variable

GRLWEAP Fundamentals
Formulas and Wave Equation
D19-42; HP 12x53; Clay and Sand

![Graph showing the relationship between Blows/0.25 m and Capacity in kN for Gates, ENR, GRLWEAP-Clay, and GRLWEAP-Sand.]
Driveability Analysis

Basically:

• **Perform a static soil analysis** – do it as accurately as possible

• **Perform wave equation analyses for different depths with** $R_u$ **from static soil analysis**

• **Plot calculated** $R_u$, blow count, maximum stresses vs. **depth**
Driveability Analysis

- Analyze a series of Bearing Graphs for different depths for SRD and/or LTSR
- Put the results in sequence so that we get predicted blow count and stresses vs pile toe penetration
- Note that, in many or most cases, shaft resistance is lower during driving (soil setup) and end bearing is about the same as long term
- In the few cases of relaxation, the toe resistance is actually higher during driving than long term
Choose first Depth to analyze

Model Hammer & Driving System

Pile Length and Model

Increase Depth

Input

Calculate Ru for first gain/loss

Analysis

Increase G/L?

Increase Depth?

Next G/L

Output

Increase Depth?

Increase G/L?
Driveability Result

GRL Engineers, Inc.

MRBS 4600, 60x2" dia., Driveability
Gain/Loss 1 at Shaft and Toe 0.500 / 2.520

GRLWEAP(TM) Version 2003

- Ult. Capacity (kN)
- Comp. Stress (MPa)
- ENTHRU (kJ)

Depth (meter)

Blow Count (blows/.25m)
Tension (MPa)
Stroke (m)
Remarks About SRD

• **WHAT IS $R_u$ **DURING DRIVING?** We call it SRD, because we lose static shaft resistance during driving.**

• **In general, we will regain static resistance by Soil Setup - primarily along shaft (maybe up to 10x in clay)**

• **During analysis we may want to analyze with full loss of setup or with partial loss of setup or with no loss of setup at all.**
For Driveability with variable setup time

- **Setup time**
  
  Defines after how much waiting time setup is gained

- **Limit Distance**
  
  Defines after how much driving distance soil setup is lost
Summary

• **The wave equation helps for equipment selection, setting preliminary driving criteria and capacity determination, if testing is not feasible.**

• **For capacity determination without measurements the factor of safety has to be greater than when doing measurements (GIGO).**
Summary

• The wave equation analysis simulates what happens in the pile due to a hammer impact.

• It calculates a relationship between capacity and blow count, or blow count vs. depth.

• The analysis model represents hammer (3 types), driving system (cushions, helmet), pile (concrete, steel, timber) and soil (at the pile-soil interface)

• GRLWEAP provides a variety of input help features (hammer and driving system data, static formulas among others).
Summary

- Good hammer performance is essential for both good productivity and a meaningful construction control by blow count.
- Wave Equation analysis results are only as good as the accuracy of the hammer efficiency and soil resistance input.
Thank you for your attention
For further information see:
www.pile.com

QUESTIONS?

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